Lesson 037
Testing Two Proportions

Friday, December 1

Two Sample Tests of Proportions

- What if we have two samples of data from independent binomial distributions?
  - We take $X \sim \text{Bin}(n, p_1)$
  - We take $Y \sim \text{Bin}(n, p_2)$
  - We assume that $X \perp Y$
- We are interested in $p_1 - p_2$.

Estimation of Differences in Proportions

- The estimator $\widehat{p}_1 - \widehat{p}_2$ is unbiased for $p_1 - p_2$.
- The variance of $\widehat{p}_1 - \widehat{p}_2$ will be

$$\text{var}(\widehat{p}_1 - \widehat{p}_2) = \frac{p_1(1 - p_1)}{n} + \frac{p_2(1 - p_2)}{m}.$$

Sampling Distribution for Proportions

- As long as the normal approximation applies for both X and Y, we can use a normal approximation for the differences in proportions.
- Suppose that we wish to test $H_0 : p_1 = p_2$ versus $H_1 : p_1 \neq p_2$.

Estimation of Differences in Proportions

- Under the null hypothesis, we can estimate a shared variance term.

$$\text{var}(\widehat{p}_1 - \widehat{p}_2) = p(1-p)\left[\frac{1}{n} + \frac{1}{m}\right].$$

- Then, we can construct the usual test statistic.

$$\frac{\widehat{p}_1 - \widehat{p}_2}{\sqrt{p(1-p)\left[n^{-1} + m^{-1}\right]}} \dot{\sim} N(0,1).$$

Tests of Specific Differences

- If we wish to test $H_0 : p_1 - p_2 = \delta_0$, for some $\delta_0 \neq 0$, this procedure does not work.
- There is not a shared variance.
- Some statisticians will use the estimated variance and appeal to the CLT, as we have for other situations.

Tests of Specific Differences

- In order to test the null hypothesis $H_0 : p_1 - p_2 = \delta_0$ versus $H_1 : p_1 - p_2 \neq \delta_0$ we can use:

$$\frac{\widehat{p}_1 - \widehat{p}_2 - \delta_0}{\sqrt{\widehat{p}_1(1 - \widehat{p}_1)/n + \widehat{p}_2(1 - \widehat{p}_2)/m}} \dot{\sim} N(0,1).$$

Suppose that you have a sample of size 10 from a single population. You estimate the mean to be 15 and the standard deviation to be 3. What is the sampling distribution under the null, $H_0 : \mu = \mu_0$?

$N(0, 1)$, assuming the population is normal. — 50%

$N(0, 1)$. — 0%

✓ t_9 assuming the population is normal. — 50%

t_9. — 0%

t_{10} assuming the population is normal. — 0%

Suppose that you have $X \sim \mathrm{Bin}(5, p)$. You wish to test $H_0 : p = 0.5$. Which of the following is true?

We can use the normal approximation to the binomial to derive a normal sampling distribution.

0%

Because the sample size is small, we need to use a t_4 distribution for the sampling distribution.

0%

Because the sample size is small, we need to use a chi-square distribution for the sampling distribution.

0%

✓ Because the sample size is small, the normal approximation will not be valid, and we cannot test this using the tools we have learned.

100%

Suppose we have data from two samples, and wish to test about their mean difference. Which of the following will typically correspond to the highest p-value, assuming that data remains the same?

✅ A standard t-test using $\min\{m-1, n-1\}$ degrees of freedom, without pooling variance.

100%

A standard t-test using the pooled variance estimator.

0%

A t-test using $\nu = \dfrac{\left(\frac{s_1^2}{n} + \frac{s_2^2}{m}\right)^2}{\frac{s_1^4/n^2}{n-1} + \frac{s_2^4/m^2}{m-1}}$ degrees of freedom, without pooling variance.

0%

A Z test, leveraging the normal approximation.

0%

Which of the following will produce the shortest confidence interval?

- A 90% confidence interval computed with a standard error of 2. — 0%
- A 95% confidence interval computed with a standard error of 2. — 0%
- ✅ A 90% confidence interval computed with a standard error of 1. — 50%
- A 95% confidence interval computed with a standard error of 2. — 50%
- A 99% confidence interval computed with a standard error of 1. — 0%

A new process is being used by a supplier. Despite assurances to the contrary, you think that the new yield (μ) is worse than the old process (with a yield of μ_0). Larger yields are preferred. What should be the null hypothesis to test this?

0

(A) $H_0 : \mu = \mu_0$.

0%

(B) $H_0 : \mu \leq \mu_0$.

0%

(C) $H_0 : \mu \geq \mu_0$.

0%

Suppose that you are considering two processes for producing a particular component. Process 1 is expensive, but is known to work well. Process 2 is cheaper, but may not produce components within spec. What is the relevant null hypothesis?

✓✓ 0

$H_0 : \mu_1 - \mu_2 = 0.$

0%

$H_0 : \mu_1 - \mu_2 \geq 0.$

0%

$H_0 : \mu_1 - \mu_2 \leq 0.$

0%

A 90% confidence interval for μ is found to be $[-2, 3]$. What is the correct interpretation of this statement?

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The probability that μ is in the interval is 0.9 (i.e., $P(\mu \in [-2, 3]) = 0.9$.)

0%

μ must take on a value within the interval (i.e., $\mu \in [-2, 3]$.)

0%

Repeated often enough, intervals computed in this way will contain μ 90% of the time.

0%

Suppose that we reject the null hypothesis at a significance level of $\alpha = 0.01$. What would have been the conclusion at a level $\alpha = 0.05$?

0

We would fail to reject the null hypothesis.

0%

We would reject the null hypothesis.

0%

We may fail to reject or reject the null hypothesis; there is not enough information to tell.

0%

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0%

Which of the following is not a suitable candidate for using a hypothesis test with paired data?

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Comparing the wear patterns of two brands of tires by installing one brand on a set of cars, driving until they wear down, then doing the same with the other brand.

0%

Comparing the efficacy of a medication by recruiting two groups which are matched on demographic and health factors.

0%

Determining whether seasonal changes impact mood by following the same group of people in the summer and winter.

0%

Comparing the strength of dominant and non-dominant hands, by measuring the distance that a weighted ball travels when pushed by each hand from a group of individuals.

0%